GET1018 Final Examination Cheat-sheet

1. Counting & Probability

1) *Fibonacci series:* $C_{n+1} = C_n + C_{n-1}$.
2) *Independent events:* $P(A \wedge B) = P(A) \cdot P(B)$.
3) *Mutually exclusive events:* $P(A \cup B) = P(A) + P(B)$.
4) When the times of experiments is large enough, frequency will converge to the underlying actual probability.
5) *Mathematical expectation:* $E(X) = \sum_{i=1}^{k} x_i p_i$ and $var(X) = E(X^2) - [E(X)]^2$.

2. Combination & Permutation

1) *Simple permutation:* $P(n,r) = n!/(n-r)!$.
2) *Simple combination:* $C(n,r) = n!/(r! \cdot (n-r)!)$.
3) *Stirling formula (approximation):* $n! = \sqrt{2\pi} \cdot \sqrt{n} \cdot (n/e)^n$ if $n \gg 1$.
4) *Euler formula:* $e^{ix} = \cos x + i \cdot \sin x$.
5) The *natural logarithm* is discovered by $e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$.
6) The value of *Pi* can be approximated by $\prod_{n=1}^{\infty} (\frac{2n}{2n-1} \cdot \frac{2n}{2n+1}) = \frac{\pi}{2}$.
7) *Binomial expansion:* $(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \cdots + \binom{n}{n} a^0 b^n$.
8) *Pascal triangle:* $C(n+1,r) = C(n,r) + C(n,r-1)$.
9) *Poker game – straight flush:* $P = \frac{4 \cdot 10}{C(52,5)}$.
10) *Poker game – four of a kind:* $P = \frac{13 \cdot (52-4)}{C(52,5)}$.
11) *Poker game – full house:* $P = \frac{13 \cdot C(4,3) \cdot 12 \cdot C(4,2)}{C(52,5)}$.
12) *Poker game – flush:* $P = \frac{4 \cdot C(13,5) - 40}{C(52,5)}$.
13) *Poker game – straight:* $P = \frac{10 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 - 40}{C(52,5)}$.
14) *Poker game – three of a kind:* $P = \frac{13 \cdot C(4,3) \cdot 48 \cdot 44}{C(52,5) \cdot 2!}$.
15) *Poker game – two pairs:* $P = \frac{13 \cdot C(4,2) \cdot 12 \cdot C(4,2) \cdot 44}{C(52,5) \cdot 2!}$.

3. Normal Distribution

1) Probability density function (PDF): $\frac{1}{\sqrt{2\pi}\sigma} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$.
2) We also have $Z = \frac{r-(nq+1/2)}{\sqrt{npq}}$ for $(p+q)^n$.

4. Game Theory

1) *Zero-sum game:* in any two-player zero-sum game, there is a certain "right" outcome. If the players follow their best strategies, the game will always have the same ending.
2) *Nash equilibrium:* a strategy profile is a Nash equilibrium if no player can do better by unilaterally changing his or her strategy.
3) If we allow mixed strategies, every game with a finite number of players (and each player has a finite number of pure strategies) will have at least one Nash equilibrium.
4) *Dominating strategy:* we say A_i dominates A_j if for each payoff in $A_i \geqslant$ each payoff in A_j and at least one payoff in $A_i >$ one payoff in A_j.
5) *Saddle point:* if there exists $\text{max-min}(A) = -\text{max-min}(B)$, then it can be used by rational players as pure strategies for the game.
6) If a stable point cannot be reached, the players may use a randomized mixed strategy to find payoff equilibrium.
7) To find the optimal solution for fixed strategies, one should express the expected payoff in the form of $(x-p) \cdot (y-q)$ and then let $x = p, y = q$.
8) *2/3 of the average:* $(0, 0, \cdots, 0)$ is the only Nash equilibrium.
9) *Diner's dilemma:* all choose the expensive dish since $x - y > (a-b)/n$.

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